MHD Free Convective Flow over an Inclined Porous Surface with Variable Suction and Radiation Effect.

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Abstract
Magneto hydrodynamic MHD free convection flow, of an incompressible viscous fluid over an inclined porous surface with variable suction and radiation effect is presented. Using the perturbation technique, solution of the set of the non-linear partial differential equations of motion, energy and diffusion, are obtained by reducing them to ordinary differential equation which are solved analytically for velocity, temperature and the concentration distributions. Effect of parameters such as magnetic field, radiation, suction, chemical reaction on the velocity, temperature, concentration, heat transfer and the skin friction distributions are discussed. The results shows that increase in Radiation parameter leads to velocity increase due to increased dominance of conduction on the flow. It is also noted that the temperature increased as a result of absorption of radiant heat from the plate. Increased Magnetic parameter retards the velocity flow as a result of a thickened boundary layer caused by Lorentz force that opposes the flow of the fluid. Increase in the chemical reaction parameter caused a reduced velocity flow and an increase in concentration due to the rise in the interfacial mass transfer and heat transfer. Increased values of suction parameter reduced the velocity of the fluid, concentration profile and heat transfer. The increase in the porosity parameter will cause a reduced resistance to the motion of the fluid which causes an increase in the velocity.

Keywords: *Free Convection, Radiation, Chemical Reaction, Inclined Plate, Porous Surface, Magneto Hydrodynamic (MHD), Variable Suction,*

1.0 Introduction

Free convection flow in the presence of a magnetic field is important and this is because it has a significant effect on the boundary layer control and the performance of many engineering devices. Practical examples include MHD power generation, plasma studies, nuclear reactor using liquid metal coolant, geothermal energy extraction etc. When the temperature of a surrounding fluid becomes high, radiation effects play an important role. In such a case, one has to examine the effect of thermal radiation and mass diffusion. Radiative heat and mass transfer flow for the cases of vertical and horizontal flat plates have been attracting many researchers due to many applications in space technology and processes involving high temperatures such as thermal energy storage, nuclear power plants and gas turbines. The researches of Kays *et al* (2004), Mebine (2007), Singh and Kumar (2011) are worthy of note. Moreover, attention has been given to boundary layer flows adjacent to inclined plate. Rao *et al.* (2014) studied about the phenomenon of heat and mass transfer in the object of extensive research Such phenomena are observed in buoyancy induced motions in the atmosphere, in bodies of water, quasi solid bodies such as earth and so on. His study showed the effect of radiation and thermo-diffusion on non-Darcy convective heat and mass transfer flow of a viscous, electrically conducting fluid through a porous medium in the presence of heat generating sources. Ramana Rendy *et al.* (2016) analyzed the effects of magneto hydrodynamic force and buoyancy on convective heat and mass transfer flow past a moving vertical porous plate in the presence of thermal radiation and chemical reaction. Srinivasacharya *et al.* (2016) studied mixed convection heat and mass transfer along a vertical plate embedded in a power-law fluid saturated Darcy porous medium with chemical reaction and radiation effects.

Ziyauddin *et al.* (2010) studied the radiation effects on unsteady MHD natural convection flow in a porous medium with conjugate heat and mass transfer past a moving inclined plate in the presence of chemical reaction, variable temperature and mass diffusion.

Venkateswarlu *et al.* (2015) analyzed the MHD unsteady flow of viscous, incompressible electrical conduction fluid over a vertical porous plate under the influence of thermal radiation and chemical reaction. A study was done by Tripathy *et al.* (2015) on the heat and mass transfer effect in a boundary layer flow of an electrically conducting viscous fluid subject to transverse magnetic field past over a moving vertical plate through porous medium in the presence of heat source and chemical reaction. Sandeep *et al*. (2012) analyzed the Magneto hydrodynamic, Radiation and chemical reaction effects on unsteady flow, heat and mass transfer characteristics in a viscous, incompressible and electrically conduction fluid over a semi-infinite vertical porous plate through porous media. The porous plate was subjected to a transverse variable suction velocity. Sugunamma *et al.* (2013) studied MHD Radiation and chemical reaction effects on unsteady flow, heat and mass transfer in a viscous, incompressible and electrically conducting fluid over a semi-infinite vertical porous plate through porous media in presence of inclined magnetic field with porous plate subjected to a transverse variable suction velocity. Isreal-Cookey *et al.* (2003) investigated the influence of viscous dissipation and radiation on unsteady MHD free-convection flow past an infinite heated vertical plate in a porous medium with time-dependent suction. Veeresh *et al.* (2015) analyzed the heat and mass transfer in MHD free convection chemically reactive and radiative flow in a moving inclined porous plate with temperature dependent heat source and joule heating using regular perturbation technique

2.0 Mathematical Formulation

We consider an unsteady free convective flow of a viscous incompressible, chemically reactive and radiative fluid over a semi-infinite plate inclined at an angle α in the vertical direction. A magnetic field of constant intensity B_0 is applied in the y-direction. The plate moves uniformly with velocity v_0 in the x-direction, the temperature $T_w > T_\infty$ and concentration $C_w > C_\infty$are conditions at the wall and ambient regions respectively. We assume a homogeneous chemical reaction and that the Boussinesq approximation is valid. Hence the governing flow equations are as follows:

$$\frac{\partial v'}{\partial y'} = 0 \tag{1}$$

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = \nu\frac{\partial^2 u'}{\partial y'} + \frac{\partial U'}{\partial t'} - \left[\mu^2\frac{\delta_c B_0^2}{\rho} + \frac{\nu}{k}\right](u' - U') + g\beta_T(T' - T_\infty)cos \propto + g\beta_C(C' - C_\infty)cos \propto \tag{2}$$

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \frac{K}{\rho C_p}\frac{\partial^2 T'}{\partial {y'}^2} + \frac{\mu}{\rho C_p}\left[\frac{\partial u'}{\partial y'}\right] - \frac{K}{\rho C_p}\frac{\partial q_r'}{\partial y'} \tag{3}$$

$$\frac{\partial C'}{\partial t'} + v'\frac{\partial C'}{\partial y'} = D\frac{\partial^2 C'}{\partial {y'}^2} - K_r'(C' - C_\infty) \tag{4}$$

The boundary conditions for the velocity (u), temperature (T) and concentration (C) fields in the above equations are:

$$u' = u'_s\,;\; T' = T'_w;\; C' = C'_w; \qquad \text{at } y = 0$$
$$u' = 0;\; T' = T'_\infty;\; C' = C'_w \qquad \text{as } y \to \infty \tag{5}$$

From the continuity equation (1)

$$v = -v_o(1 + \varepsilon A e^{i\omega t}) \tag{6}$$

Such that $\varepsilon A \ll 1$ and the negative sign indicates that the suction velocity is towards the porous surface.

We adopt the Rosseland approximation for the radiative heat flux, q'_r. That is:

$$q'_r = -\frac{4\delta'}{3k'}\frac{\partial T'}{\partial y'} = -\frac{4\delta'}{3k'}\nabla T' \tag{7a}$$

where δ' is the Stefan-Boltzmann constant and k' is the Rosseland mean absorption coefficient. Assuming that the temperature differences within the flow are sufficiently small that T' may be expressed as a linear function of temperature, then

$$T' = 4T_O^3 T - 3T'_O$$

This implies that:

$$q'_r = -\frac{16\delta' T_O^3}{3k'}\frac{\partial T}{\partial y} \tag{7b}$$

In order to write the governing equations and boundary conditions in dimensionless form, the following non-dimensional quantities are introduced

$$y = \frac{v'_o y'}{\nu};\; u = \frac{u'}{V_O}\,;\; w = \frac{\nu w'}{v_o'^2}\,;\, t = \frac{v_o'^2 t'}{\nu}\,; U = \frac{U'}{V_O}\,;\; \theta = \frac{T' - T_\infty}{T'_w - T_\infty}\,; C = \frac{C' - C_\infty}{C'_w - C_\infty} \tag{8}$$

$$M^2 = \frac{\nu\mu^2\delta_c H_O'^2}{\rho v_o'^2}\,;\; \chi^2 = \frac{\nu^2}{K v_o'^2}\,;\; G_{r_T} = \frac{g\nu\beta_T\theta(T_w' - T_\infty)}{V_O v_o'^2}\,;\; G_{r_c} = \frac{g\nu\beta_C C(C_w' - C_\infty)}{V_O v_o'^2}\,; \nu = \frac{\mu}{\rho}; P_r = \frac{\mu C_p}{k}\,;$$

$$R^2 = \frac{16\nu\delta' T_O^3}{3k'}\,;\; Ec = \frac{V_O^2}{C_p(T'_w - T_\infty)};\; S_c = \frac{\nu}{D} \qquad ,\; u_s = \frac{u'_s}{V_O}$$

The momentum, energy and diffusion equation in dimensionless form is written as

$$\frac{\partial u}{\partial t} - \left(1 + \varepsilon A e^{i\omega t}\right)\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + \frac{\partial U}{\partial t} - [(M^2 + \chi^2)(u - U)] + G_{r_T}\theta cos \propto + G_{r_c} C cos \propto \tag{9}$$

$$Pr\frac{\partial \theta}{\partial t} - Pr\left(1 + \varepsilon A e^{i\omega t}\right)\frac{\partial \theta}{\partial y} = (1 + R^2)\frac{\partial^2 \theta}{\partial y^2} + P_r Ec\left(\frac{\partial u}{\partial y}\right)^2 \tag{10}$$

$$Sc\frac{\partial C}{\partial t} - Sc\left(1 + \varepsilon A e^{i\omega t}\right)\frac{\partial C}{\partial y} = \frac{\partial^2 C}{\partial y^2} - ScK_r C \tag{11}$$

$$u = u_s\,;\; \theta = 1;\; C = 1 \qquad \text{at } y = 0 \tag{12}$$
$$u \to 0,\; \theta \to 0,\; C \to 0 \qquad \text{as } y \to \infty \tag{13}$$

In the above equations u and v are the velocity components, x and y are the Cartesian coordinates, α is the angle of inclination in the vertical direction of the semi-infinite moving plate. t' is the time, T'_w is the temperature at the wall, T'_∞ is the reference temperature and g is the acceleration due to gravity. v_o is the suction velocity, H_O^2 is the constant magnetic field, K is the porosity parameter, C_p is the specific heat capacity and M^2 is the magnetic parameter. $G_{r_T}$is Grashof temperature number, Pr is the Prandtle number, R^2 is the radiation parameter, U_o is the mean velocity of U' is the free stream velocity. q'_r is the radiation heat flux, Ec is the Eckert number, ε is the small positive constant, ρ is the density, β_T is the coefficient of volume expansion due to temperature and β_C is the coefficient of volume expansion due to concentration. χ^2 is the Darcy number δ is the radiation absorption, k is the coefficient of thermal conduction, ω is the free stream frequency oscillation, μ is the permeability, ϑ_c is the electrical conductivity, G_{r_c} is the modified Grashof number, K_r is the chemical reaction parameter, θ is the temperature and C_w is the concentration at the wall.

3.0 Method of Solution

Equations (2) – (4) are coupled nonlinear partial differential equations that is solved analytically and then reduced to a set of ordinary differential equations with the expressions for velocity (u), temperature (θ) and concentration (C) of the fluid in dimensionless form as follows:

$$u(y,t) = u_o(y) + \varepsilon e^{i\omega t}u_1 + 0(\varepsilon^2) \quad (14)$$
$$\theta(y,t) = \theta_o(y) + \varepsilon e^{i\omega t}\theta_1 + 0(\varepsilon^2) \quad (15)$$
$$C(y,t) = C_o(y) + \varepsilon e^{i\omega t}C_1 + 0(\varepsilon^2) \quad (16)$$

Substituting equation (14) to (16) in the set of equations (9), (10) and (11) and equating non – harmonic and harmonic terms and neglecting the higher order terms of $0(\varepsilon^2)$, the following set of ordinary differential equations are obtained with their boundary conditions.

$$u_0^{II} + u_0^{I} - (M^2 + \chi^2)u_0 = -(M^2 + \chi^2) - G_{r_T}\theta_0 cos \propto - G_{r_c}C_0 cos \propto \quad (17)$$
$$(1 + R^2)\theta_0^{II} + P_r\theta_0^{I} = P_rE_cu_0^{I2} \quad (18)$$
$$C_0^{II} + S_cC_0^{I} - S_cK_rC_o = 0 \quad (19)$$
$$u_1^{II} + u_1^{I} - (M^2 + \chi^2 + i\omega)u_1 = -Au_0^{I} - (M^2 + \chi^2 + i\omega) - G_{r_T}\theta_1 cos \propto - G_{r_c}C_1 cos \propto \quad (20)$$
$$(1 + R^2)\theta_1^{II} + P_r\theta_1^{I} - P_ri\omega\theta_1 = -P_rA\theta_0^{I} - 2P_rE_cu_0^{I}u_1^{I} \quad (21)$$
$$C_1^{II} + S_cC_1^{I} - (S_cK_r + S_ci\omega)C_1 = -S_cAC_0^{I} \quad (22)$$

Boundary conditions

$$u_0 = u_s; u_1 = 0;\ \theta_0 = 1;\ ,\theta_1 = 1;\ C_0 = 1;\ C_1 = 1 \qquad \text{at } y = 0 \quad (23)$$
$$u_0 \to 0; u_1 \to 0;\ \theta_0 \to 0;\ ,\theta_1 \to 0;\ C_0 \to 0;\ C_1 \to 0 \qquad \text{as } y \to \infty \quad (24)$$

To solve the nonlinear coupled equations (17) – (22) we assume that the dissipation parameter (Eckert number $Ec \ll 1$) is small, and therefore advance an asymptotic expansion for the flow temperature and velocity as follows using the boundary condition in equation (23 – 24):

$$u_0(y) = u_{o1}(y) + E_cu_{o2}(y)$$
$$\theta_0(y) = \theta_{o1}(y) + E_c\theta_{o2}(y) \quad (25)$$
$$u_1(y) = u_{11}(y) + E_cu_{12}(y)$$
$$\theta_1(y) = \theta_{11}(y) + E_c\theta_{12}(y)$$

Substituting Eq. (25) into Eq. (17) – (24) we obtain the following sequence of approximation

$$u_{01}^{II} + u_{01}^{I} - (M^2 + \chi^2)u_{01} = -(M^2 + \chi^2) - G_{r_T}\theta_{01} cos \propto - G_{r_c}C_0 cos \propto \quad (26)$$
$$(1 + R^2)\theta_{01}^{II} + P_r\theta_{01}^{I} = 0 \quad (27)$$
$$u_{02}^{II} + u_{02}^{I} - (M^2 + \chi^2)u_{02} = -G_{r_T}\theta_{02} cos \propto \quad (28)$$
$$(1 + R^2)\theta_{02}^{II} + P_r\theta_{02}^{I} = -P_ru_{01}^{I\,2} \quad (29)$$

with boundary conditions:

$$u_{01} = u_s, \theta_{01} = 1; u_{02} = 0 = \theta_{02} \qquad \text{at } y = 0 \quad (30)$$
$$u_{01} \to 0;\ \theta_{01} \to 0; u_{02} \to 0;\ \theta_{02} \to 0; \qquad \text{as } y \to \infty \quad (31)$$

$$u_{11}^{II} + u_{11}^{I} - (M^2 + \chi^2 + i\omega)u_{11} = -Au_{01}^{I} - (M^2 + \chi^2 + i\omega) - G_{r_T}\theta_{11} cos \propto - G_{r_c}C_1 cos \propto \quad (32)$$
$$(1 + R^2)\theta_{11}^{II} + P_r\theta_{11}^{I} - P_ri\omega\theta_{11} = -P_rA\theta_{01}^{I} \quad (33)$$

$$u_{12}'' + u_{12}' - (M^2 + \chi^2 + i\omega)u_{12} = -Au_{02}' - G_{r_T}\theta_{12}cos \propto \quad (34)$$
$$(1 + R^2)\theta_{12}'' + P_r\theta_{12}' - P_r i\omega\theta_{12} = -P_r A\theta_{02}' - 2P_r u_{11}' u_{01}' \quad (35)$$

Boundary condition

$$u_{11} = 0 = \theta_{11} = 0; u_{12} = 0 = \theta_{12}; \quad \text{at } y = 0 \quad (36)$$
$$u_{11} \to 0;\ \theta_{11} \to 0; u_{12} \to 0;\ \theta_{12} \to 0; \quad \text{as } y \to \infty \quad (37)$$

where $N_1 = (M^2 + \chi^2); N_2 = M^2 + \chi^2 + i\omega$ solving equations (26) – (29) with the boundary conditions (30) – (31) and equation (32) – (35) satisfying the boundary condition (36) – (37), we have

$$u(y,t) = D_1e^{-m_3y} - D_2e^{-m_2y} - D_3e^{-m_1y} - 1 + E_c\big[D_{11}e^{-m_5y} - D_{12}e^{-m_4y} + D_{13}e^{-2m_3y} + D_{14}e^{-2m_2y} + D_{15}e^{-2m_1y} - D_{16}e^{-(m_2+m_3)y} - D_{17}e^{-(m_1+m_3)y} - D_{18}e^{-(m_1+m_2)y}\big] + \varepsilon e^{i\omega t}\{D_{23}e^{-m_8y} - D_{24}e^{-m_7y} - D_{26}e^{-m_6y} + D_{28}e^{-m_3y} - L_1e^{-m_2y} - L_2e^{-m_1y} + 1 + E_c\big[D_{57}e^{-m_{10}y} + D_{58}e^{-m_5y} - D_{59}e^{-m_4y} + D_{60}e^{-2m_3y} + D_{61}e^{-2m_2y} + D_{62}e^{-2m_1y} - D_{63}e^{-(m_2+m_3)y} - D_{64}e^{-(m_1+m_3)y} - D_{65}e^{-(m_1+m_2)y} - D_{66}e^{-m_9y} - D_{67}e^{-m_4y} + D_{68}e^{-2m_3y} + D_{69}e^{-2m_2y} + D_{70}e^{-2m_1y} - D_{71}e^{-(m_2+m_3)y} - D_{72}e^{-(m_1+m_3)y} + D_{73}e^{-(m_1+m_2)y} + D_{74}e^{-(m_3+m_8)y} + D_{75}e^{-(m_2+m_8)y} - D_{76}e^{-(m_1+m_8)y} - D_{77}e^{-(m_3+m_7)y} + D_{78}e^{-(m_2+m_7)y} + D_{79}e^{-(m_1+m_7)y} - D_{80}e^{-(m_3+m_6)y} + D_{81}e^{-(m_2+m_6)y} + D_{82}e^{-(m_1+m_6)y}\big]\} \quad (38)$$

$$\theta(y,t) = e^{-m_2y} + E_c\big[D_4e^{-m_4y} - D_5e^{-2m_3y} - D_6e^{-2m_2y} - D_7e^{-2m_1y} + D_8e^{-(m_2+m_3)y} + D_9e^{-(m_1+m_3)y} - D_{10}e^{-(m_1+m_2)y}\big] + \varepsilon e^{i\omega t}\{D_{21}e^{-m_7y} + D_{22}e^{-m_2y} + E_c\big[D_{31}e^{-m_9y} + D_{32}e^{-m_4y} - L_3e^{-m_3y} - L_4e^{-m_2y} - L_5e^{-m_1y} + L_6e^{-(m_2+m_3)y} + L_7e^{-(m_1+m_3)y} - L_8e^{-(m_1+m_2)y} - D_{39}e^{-(m_3+m_8)y} + D_{40}e^{-(m_2+m_8)y} + D_{41}e^{-(m_1+m_8)y} + D_{42}e^{-(m_3+m_7)y} - D_{43}e^{-(m_2+m_7)y} - D_{44}e^{-(m_1+m_7)y} + D_{45}e^{-(m_3+m_6)y} - D_{46}e^{-(m_2+m_6)y} - D_{47}e^{-(m_1+m_6)y}\big]\} \quad (39)$$

$$C(y,t) = e^{-m_1y} + \varepsilon e^{i\omega t}\{D_{19}e^{-m_6y} + D_{20}e^{-m_1y}\} \quad (40)$$

The physical quantities of interest are the wall shear stress τ_w is given by

$$\tau_w = \mu\frac{\partial u^I}{\partial y^I} = \rho v_0^2 u^I(0)$$

The local skin friction factor

$$C_{fx} = \frac{\tau_w}{\rho v_0^2} = u^I(0) = -m_3D_1 + m_2D_2 + m_1D_3 + E_c[-m_5D_{11} + m_4D_{12} - 2m_3D_{13} - 2m_2D_{14} - 2m_1D_{15} + (m_2+m_3)D_{16} + (m_1+m_3)D_{17} + (m_1+m_2)D_{18}] + \varepsilon[-m_8D_{23} + m_7D_{24} + m_6D_{26} - m_3D_{28} + m_2L_1 + m_1L_3 + E_c(-m_{10}D_{57} - m_5D_{58} + m_4L_9 - 2m_3L_{10} - 2m_2L_{11} - 2m_1L_{12} - (m_2+m_3)L_{13} + (m_1+m_3)L_{14} + (m_1+m_2)L_{15} + m_9D_{66} - (m_3+m_8)D_{74} - (m_2+m_8)D_{75} + (m_1+m_8)D_{76} + (m_3+m_7)D_{77} - (m_2+m_7)D_{78} - (m_1+m_7)D_{79} + (m_3+m_6)D_{80} - (m_2+m_6)D_{81} - (m_1+m_6)D_{82})] \quad (41)$$

$$\frac{N_{ux}}{R_{ex}} = \frac{\partial\theta}{\partial y} = \theta^I(0) = -m_2 + E_c[-m_4D_4 + 2m_3D_5 + 2m_2D_6 + 2m_1D_7 - (m_2+m_3)D_8 = -(m_1+m_3)D_9 + (m_1+m_2)D_{10}] + \varepsilon[-m_7D_{21} - m_2D_{22} + E_c(-m_9D_{31} - m_4D_{32} + 2m_2L_3 + 2m_1L_5 - (m_2+m_3)L_6 - (m_1+m_3)L_7 + (m_1+m_2)L_8 + (m_3+m_8)D_{39} + (m_2+m_8)D_{40} - (m_1+m_8)D_{41} - (m_3+m_7)D_{42} + (m_2+m_7)D_{43} + (m_1+m_7)D_{44} - (m_3+m_6)D_{45} + (m_2+m_6)D_{46} + (m_1+m_6)D_{47})] \quad (42)$$

The local surface mass flux is given by

$$\frac{Sh_x}{R_{ex}} = -\frac{\partial C}{\partial y} = C^I(0) = -m_1 - \varepsilon(m_6D_{19} + m_1D_{20}) \quad (43)$$

4.0 Results and Discussion

For various parameter values, we have shown plots indicating the effects of the major parameters governing flow. Consequently, we present our interpretation of the results. It is observed in Figure 4.1 that increase in the radiation parameter leads to increase in the velocity. This is as a result of increased dominance of conduction due to the increase in radiation. There is therefore increase in the buoyancy forces in the boundary layer which ultimately increases the fluid velocity. The effect of the magnetic field is shown in Figure 4.2. The profile shows that increase in the magnetic field retards the flow velocity. Physically this is because the Lorentz force thickens the boundary layer and therefore reduces the flow velocity. In figure 4.3 we see that the velocity of the fluid decreases with increasing value of the suction. The suction is towards the plate, this in effect has reducing consequences on the flow velocity. It is observed in Figure 4.4 that increase in the chemical reaction parameter decreases the velocity. The effect is more pronounced in the vicinity of the boundary layer. Figure 4.5 illustrates the effect of the Grashof number on the velocity. The fluid velocity increases as the Grashof number increases. This is because the plate is being cooled with convection currents which results in the increase of the velocity. In figure 4.6 we notice similar effect when the modified Grashof number is increased. The effect of the porosity parameter is depicted in figure 4.7. It shows that increase in the porosity term reduces the resistance to motion of the fluid, hence the velocity is increased. The profile of the variation in the inclination of the plate is shown in figure 4.8. The velocity decreases with increasing value of the inclination parameter α. At $\alpha = 10^o$, the velocity profile peaks up close to the plate and thereafter reduces at the free stream. At $\alpha \geq 20^o$, the velocity is significantly reduced, showing downward trend all through. It is observed in Figure 4.9 that the fluid temperature increases with increase in thermal radiation. This physically shows that there is absorption of radiant heat from the vertical plate and therefor increases the fluid temperature. Figure 4.10 illustrates the effect of the Grashof number o the temperature. The temperature of the fluid decreases with increasing values of the Grashof number. Physically this is true as the Grashof number is associated with cooling of the plate. Chemical reaction parameter is important in this flow model especially in the concentration profile. It is observed in Figure 4.11 that the concentration increases with increase in the chemical reaction. The chemical reaction raises the rate of interfacial mass transfer and reduces local concentration. This leads to increase in concentration gradient/flux. The concentration profile decrease as the suction parameter is increased. In Figure 4.12 the effect of porosity on heat transfer is illustrated. It is observed that the heat transfer increases with porosity and further with increase in the Grashof number. Increase in the suction and magnetic parameter both show decrease in the rate of heat transfer as in Figure 4.13. Figure 4.14 shows the effect of magnetic field on the skin friction with variable radiation. It is observed from the profile that increase in the magnetic parameter increases the skin friction. The variability on the radiation parameter shows no remarkable influence on the skin friction with respect to the effect of magnetic field.

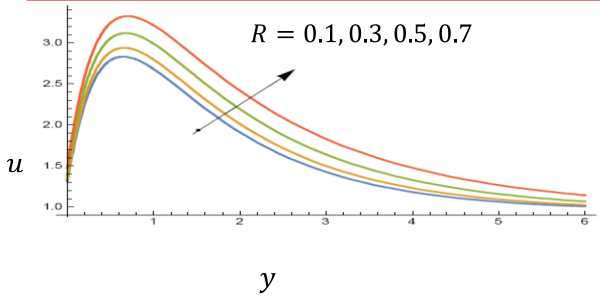

Figure 4.1: Velocity Profile with Variation of Radiation Parameter R for $A = 1, K_r = 2, M = 2, P_r = 0.71, G_r = 15, G_c = 10, E_c = 0.01, S_c = 0.22, \chi = 0.2, \alpha = 10, \omega = 1, t = 1, \varepsilon = 0.1,\ u_s = 1$

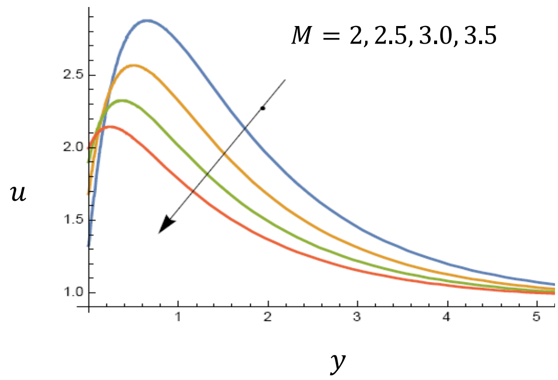

Figure 4.1: Velocity Profile with Variation of Radiation Parameter R for $A = 1, K_r = 2, R = 0.2, P_r = 0.71, G_r = 15,\ G_c = 10, E_c = 0.01, S_c = 0.22, \chi = 0.2, \alpha = 10, \omega = 1, t = 1, \varepsilon = 0.1,\ u_s = 1$

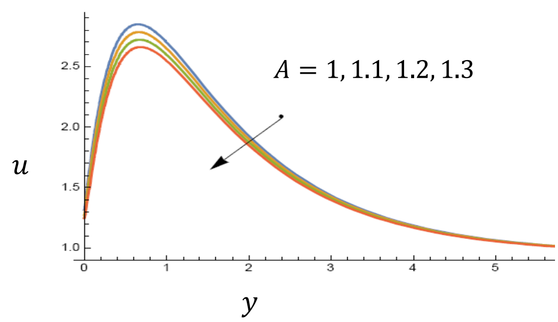

Figure 4.3: Velocity Profile with Variation of Suction Parameter A for $R = 0.1, K_r = 2, M = 2, P_r = 0.71, G_r = 15, G_c = 10, E_c = 0.01, S_c = 0.2, \chi = 0.2, \alpha = 10, \omega = 1, t = 1, \varepsilon = 0.1,\ u_s = 1$

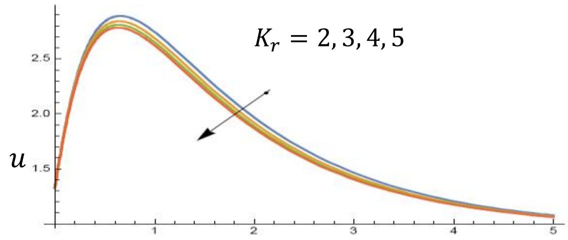

Figure 4.4: Velocity Profile with Variation of Chemical Parameter K_r for $R = 0.2, A = 1,\ M = 2, P_r\ 0.71, G_r = 15, G_c = 10, E_c = 0.01, S_c = 0.2, \chi = 0.2, \alpha = 10, \omega = 1, t = 1, \varepsilon = 0.1,\ u_s = 1$

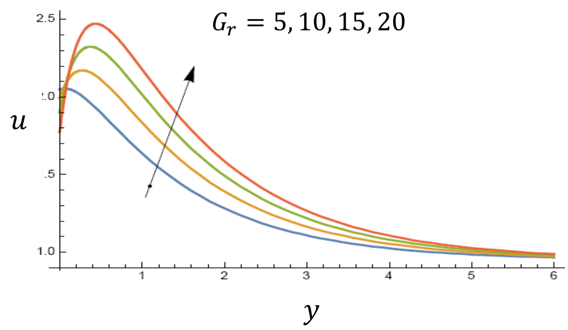

Figure 4.5: Velocity Profile with Variation of Grashof Number Parameter G_r for $R = 0.2, A = 1, K_r = 2,\ M = 3, P_r = 0.71, G_c = 10, E_c = 0.01, S_c = 0.22, \chi = 0.2, \alpha = 10,\ \omega = 1, t = 1, \varepsilon = 0.1,\ u_s = 1$

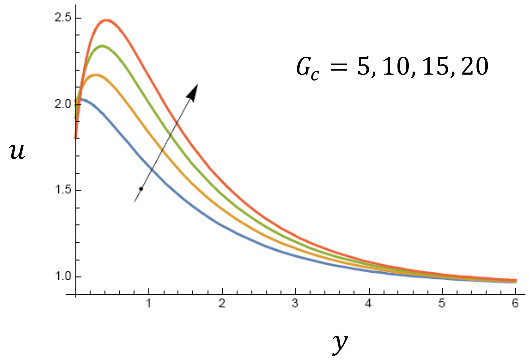

Figure 4.6: Velocity Profile with Variation of Grashof Number Parameter G_c for $R = 0.2, A = 1, K_r = 2,\ M = 3, P_r = 0.71, G_r = 15, E_c = 0.01, S_c = 0.22, \chi = 0.2, \alpha = 10, \omega = 1, t = 1, \varepsilon = 0.1,\ u_s = 1$

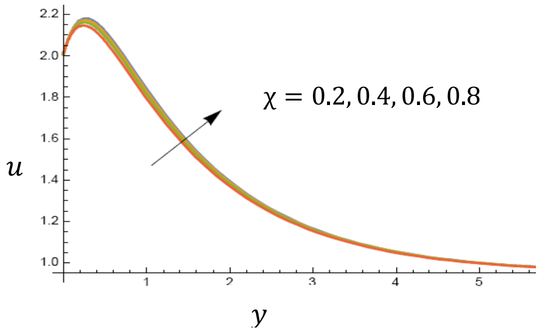

Figure 4.7: Velocity Profile with Variation of Porosity Parameter χ for $R = 0.2, A = 1, K_r = 2, M = 3, P_r = 0.71, G_r = 15, G_c = 10, E_c = 0.02, S_c = 0.22, \alpha = 10, \omega = 1, t = 1, \varepsilon = 0.1, u_s = 1$

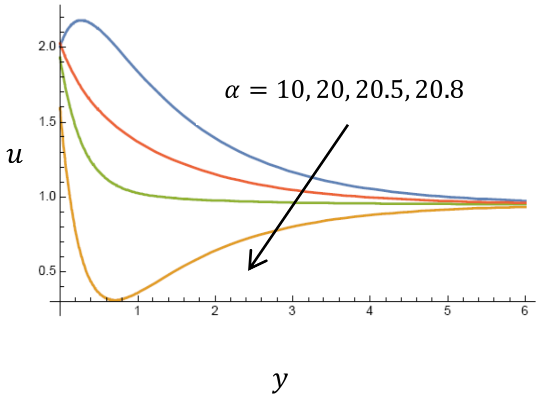

Figure 4.8: Velocity Profile with Variation of Angular Parameter α for $R = 0.2, A = 1, K_r = 2, M = 3, P_r = 0.71, G_r = 15, G_c = 10, E_c = 0.02, S_c = 0.22, \chi = 0.2, \omega = 1, t = 1, \varepsilon = 0.1, u_s = 1$

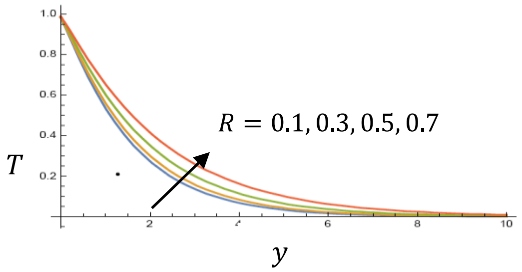

Figure 4.9: **Temperature Profile with Variation of Radiation Parameter *R*** for $A = 1, K_r = 2, M = 2, P_r = 0.71, G_r = 15, G_c = 10, E_c = 0.01, S_c = 0.2, \chi = 0.2, \alpha = 10, \omega = 1, t = 1, \varepsilon = 0.1, u_s = 1$

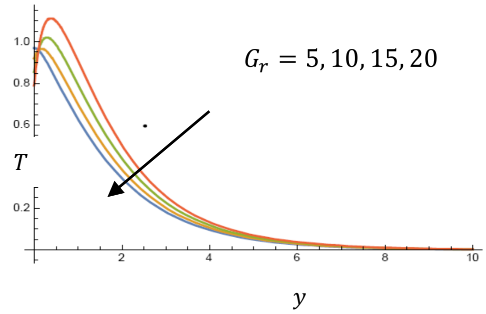

Figure 4.10: Temperature Profile with Variation of Grashof Number Parameter G_r for $R = 0.3, A = 1, K_r = 2, M = 2, P_r = 0.71, G_c = 10, E_c = 0.1, S_c = 0.2, \chi = 0.2, \alpha = 10, \omega = 1, t = 1, \varepsilon = 0.1, u_s = 1$

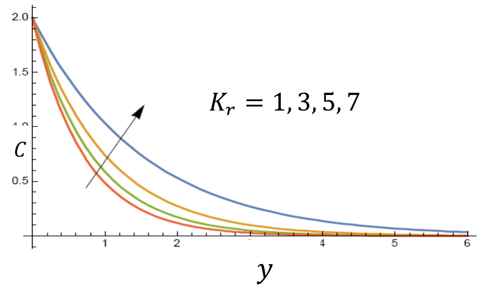

Figure 4.11: **Concentration Profile with Variation of Chemical Parameter K_r** for $A = 1, P_r = 0.71, S_c = 0.22, \omega = 0.01, t = 0.2, \varepsilon = 1, u_s = 1$

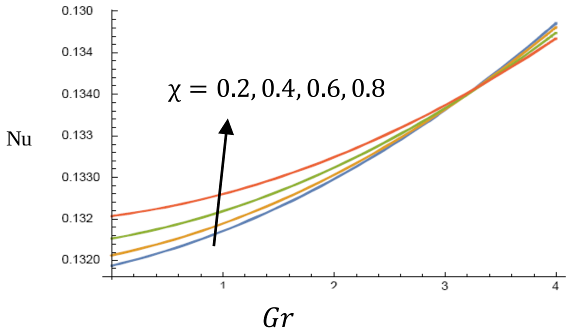

Figure 4.12: **Heat Transfer Profile with Variation of Porosity Parameter χ** for $R = 2, A = 1, K_r = 2, M = 3, P_r = 0.71, E_c = 0.2, S_c = 0.2, \alpha = 10, \omega = 1, t = 1, \varepsilon = 0.1, u_s = 1$

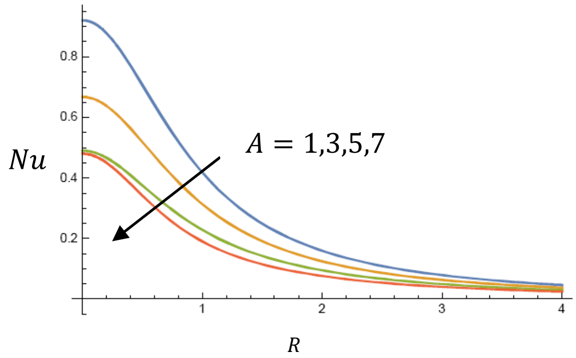

Figure 4.13: Heat transfer Profile with Variation of Suction Parameter A for $K_r = 2, M = 2, P_r = 0.71, G_r = 15, G_c = 10, E_c = 0.01, S_c = 0.2, \chi = 0.2, \alpha = 10, \omega = 1, t = 1, \varepsilon = 0.1,\ u_s = 1$

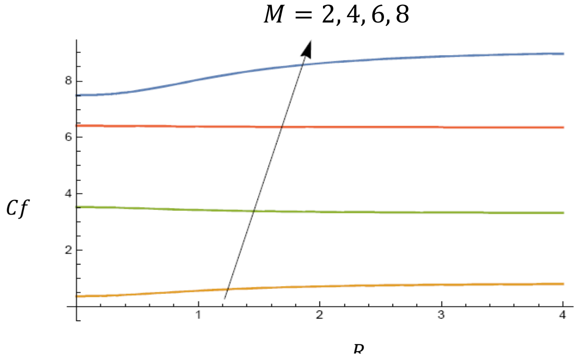

Figure 4.14: Heat Transfer Profile with Variation of Magnetic Parameter M for $A = 1,\ \ K_r = 2,\ \ \ P_r = 0.71, G_r = 15, G_c = 10, E_c = 0.01, S_c = 0.2, \chi = 0.2, \alpha = 10, \omega = 1, t = 1, \varepsilon = 0.1\ u_s = 1$

6.0 Conclusion

In conclusion therefore, the radiation and chemical reaction effect on MHD free convective flow over an inclined porous plate with variable suction and other parameter effect on the flow are summarized as follows, Radiation parameter increase led to velocity increase as a result of increased dominance of conduction, temperature increased as a result of absorption of radiant heat from the plate that is inclined and an increase on heat transfer but no effect on the Skin friction. Increased Magnetic parameter retards the velocity flow as a result of a thickened boundary layer caused by Lorentz force that opposes the flow of the fluid. Reduction of the temperature of the fluid at the wall of the plate that is inclined and a decrease in the rate of heat transfer but an increase in Skin friction. Increase in the chemical reaction parameter caused a reduced velocity flow but an increase in concentration due to the rise in the interfacial mass transfer and heat transfer. Increased values of suction parameter reduced the velocity flow, concentration profile and heat transfer. The increase in the porosity parameter led to a reduced resistance to the motion of the fluid and therefore an increase in the velocity. Increase in Grashof number cooled the plate due to convective current with increase in both velocity and heat transfer and decreased in both skin friction and temperature of the fluid while skin friction increased due to an increase in Schmidt number.

References

Israel-Cookey C., Ogulu A. and Omubo-Pepple V.B. (2003) Influence of viscous dissipation and radiation on unsteady MHD free-convection flow past an infinite heated vertical plate in a porous medium with time-dependent suction, International Journal of Heat and Mass Transfer 46, 2305–2311

Kays W; Crawford M. and Weigand, B (2004). Convective Heat and Mass Transfer, 4E. McGraw-Hill Professional

Mebine P (2007) Thermosolutal MHD flow with radiative heat transfer past an oscillating plate. *Advances in Theoretical and Applied Mathematics*, 2, 3, pp 217-231

Ramana Reddy G. V., N. BhaskarBeddy and R.S.R. Gorla (2016) Radiation and chemical reaction effects on MHD flow Along a moving vertical porous plate Int. J. Of Applied Mechanics and Engineering, vol.21, no.1, pp.157-168

Rao S.R., Rao U.R., and Rao, D.R.V.P. (2014) Mixed convective heat and mass transfer flow of a viscous fluid in a vertical channel with thermal radiation and soret effect, International Journal of Mathematical Archive 5(3), 59-70.

Sandeep N., Vijaya Bhaskar A. and Reddy V. S (2012) Effect of Radiation and Chemical Reaction on Transient MHD Free Convective Flow over a Vertical Plate through Porous Media. S.V.U College of Mathematical and Physical Sciences, ISSN 2224-7467 (Paper) ISSN 2225-0913 (Online)Vol 2,

Singh K. D. and Kumar R. (2011) “Fluctuating Heat and Mass Transfer on Unsteady MHD Free Convection Flow of Radiating and Reacting Fluid past a Vertical Porous Plate in Slip- Flow Regime,” vol. 4, no. 4, pp. 101–106.

Srinivasacharya D, Swamy Reddy G, (2016) Chemical reaction and radiation effects on mixedconvection heat and mass transfer over a vertical plate in power-law fluid saturated porous medium. Journal of the Egyptian Mathematical Society 24, 108–115.

Sugunamma V., Sandeep N., P. Mohan Krishna,Ramana Bahunadam. (2013) Inclined Magnetic field and Chemical Reaction Effects on Flow over a Semi Infinite Vertical Porous Plate through Porous Medium Communications in Applied Sciences. Vol. 1, Number 1, 1-24

Tripathy R.S., Dash G.C., Mishra S.R. and Baag S. (2015) Chemical reaction effect on MHD free convective surface over a moving vertical plate through porous medium. Alexandria Engineering Journal 54, 673–679

Veeresh C., Praveen S. V. D. and Varma K. (2015) Heat and mass transfer in MHD free convection chemically reactive and radiative flow in a moving inclined porous plate with temperature dependent heat source and joule heating, International Journal of Management, Information Technology and Engineering. Vol. 3, Issue 11, 63-74

Venkateswarlu M, and Pdman P (2015) Unsteady MHD free convective Heat and Mass Transfer in a Boundary Layer Flow Past a vertical Permeable plate with Thermal Radiation and Chemical Reaction. Internal Conference of Computational Heat and Mass Transfer, Procedia Engineering 127, 791 – 799

Ziyauddin, Kumar M (2010) Radiation effect on unsteady MHD heat and mass transfer flow on a moving inclined porous heated plate in the presence of chemical reaction. Int J Math Modell Simul App l3: 155–163.

Appendix

$M^2 = \frac{\nu\mu^2\delta_C H_O'^2}{\rho v_o'^2}$; $\chi^2 = \frac{\nu^2}{K v_o'^2}$; $G_{r_T} = \frac{g\nu B_T\theta(T_w'-T_\infty)}{V_O v_o'^2}$; $G_{r_c} = \frac{g\nu B_C C(C_w'-C_\infty)}{V_O v_o'^2}$; $P_r = \frac{\mu C_p}{k}$; $R^2 =$

$\frac{16\nu\delta' T_O^3}{3k'}$; $E_C = \frac{V_O^2}{C_p(T_w'-T_\infty)}$; $S_c = \frac{\nu}{D}$; $(M^2+\chi^2) = N_1; N_2 = M^2+\chi^2+i\omega$

$m_1 = \frac{S_c+\sqrt{(S_c{}^2+4S_cK_r)}}{2}$; $m_2 = \frac{P_r}{(1+R^2)} = m_4$; $m_3 = \frac{1+\sqrt{(1^2+4N_1)}}{2} = m_5$; $m_6 = \frac{S_c+\sqrt{\left(S_c{}^2+4(S_cK_r+S_ci\omega)\right)}}{2}$

$m_7 = \frac{P_r+\sqrt{\left(P_r{}^2+4P_ri\omega(1+R^2)\right)}}{2(1+R^2)} = m_9$; $m_8 = \frac{1+\sqrt{(1^2+4N_2)}}{2} = m_{10}$

$D_1 = D_2 + D_3 - u_s$; $D_2 = \frac{G_{r_T}\cos\propto}{m_2^2-m_2-N_1}$; $D_3 = \frac{G_{r_c}\cos\propto}{m_1^2-m_1-N_1}$; $K=1$; $D_4 = D_5+D_6+D_7-D_8-D_9+D_{10}$

$D_5 = \frac{P_r m_3^2 D_1^2}{[4(1+R^2)m_3^2-2P_rm_3]}$; $D_6 = \frac{P_r m_2^2 D_2^2}{[4(1+R^2)m_2^2-2P_rm_2]}$; $D_7 = \frac{P_r m_1^2 D_3^2}{[4(1+R^2)m_1^2-2P_rm_1]}$; $D_8 = \frac{2P_rm_2m_3D_1D_2}{[(1+R^2)(m_2+m_3)^2-P_r(m_2+m_3)]}$; $D_9 = \frac{2P_rm_1m_3D_1D_3}{[(1+R^2)(m_1+m_3)^2-P_r(m_1+m_3)]}$; $D_{10} = \frac{2P_rm_1m_2D_2D_3}{[(1+R^2)(m_1+m_2)^2-P_r(m_1+m_2)]}$; $D_{11} = D_{12}-D_{13}-D_{14}-D_{15}+D_{16}+D_{17}-D_{18}$

$D_{12} = \frac{G_{r_T}D_4\cos\propto}{[m_4^2-m_4-N_1]}$; $D_{13} = \frac{G_{r_T}D_5\cos\propto}{[m_3^2-m_3-N_1]}$; $D_{14} = \frac{G_{r_T}D_6\cos\propto}{[m_2^2-m_2-N_1]}$

$D_{15} = \frac{G_{r_T}D_7\cos\propto}{[m_1^2-m_1-N_1]}$; $D_{16} = \frac{G_{r_T}D_8\cos\propto}{[(m_2+m_3)^2-(m_2+m_3)-N_1]}$; $D_{17} = \frac{G_{r_T}D_8\cos\propto}{[(m_1+m_3)^2-(m_1+m_3)-N_1]}$

$D_{18} = \frac{G_{r_T}D_8\cos\propto}{[(m_1+m_2)^2-(m_1+m_2)-N_1]}$; $D_{19} = 1-D_{20}$; $D_{20} = \frac{S_cAm_1}{[m_1{}^2-S_cm_1-(S_cK_r+S_ci\omega)]}$; $D_{21} = -D_{22}$

$D_{22} = \frac{P_rAm_2}{[(1+R^2)m_2{}^2-P_rm_2-P_ri\omega]}$; $D_{23} = D_{24}+L_1+L_2+D_{26}-D_{28}-1$

$D_{24} = \frac{G_{r_T}D_{21}\cos\propto}{[m_7{}^2-m_1-N_2]}$; $D_{25} = \frac{G_{r_T}D_{22}\cos\propto}{[m_2{}^2-m_2-N_2]}$; $D_{26} = \frac{G_{r_C}D_{19}\cos\propto}{[m_6{}^2-m_6-N_2]}$; $D_{27} = \frac{G_{r_C}D_{20}\cos\propto}{[m_1{}^2-m_1-N_2]}$;

$D_{28} = \frac{Am_3D_1}{[m_3{}^2-m_3-N_2]}$; $D_{29} = \frac{Am_2D_2}{[m_2{}^2-m_2-N_2]}$; $D_{30} = \frac{Am_1D_3}{[m_1{}^2-m_1-N_2]}$; $J=-1$

$D_{31} = -D_{32}+L_3+L_4+L_5-L_6-L_7+L_8+D_{39}-D_{40}-D_{41}-D_{42}+D_{43}+D_{44}-D_{45}+D_{46}+D_{47}$

$D_{32} = \frac{P_rAm_4D_4}{[(1+R^2)m_4^2-P_rm_4-P_ri\omega]}$; $D_{33} = \frac{2P_rAm_3D_5}{[(1+R^2)m_3^2-2P_rm_3-P_ri\omega]}$; $D_{34} = \frac{2P_rAm_2D_6}{[(1+R^2)m_2^2-2P_rm_2-P_ri\omega]}$; $D_{35} = \frac{2P_rAm_1D_7}{[(1+R^2)m_1^2-2P_rm_1-P_ri\omega]}$; $D_{36} = \frac{P_rA(m_2+m_3)D_8}{[(1+R^2)(m_2+m_3)^2-P_r(m_2+m_3)-P_ri\omega]}$; $D_{37} = \frac{P_rA(m_1+m_3)D_9}{[(1+R^2)(m_1+m_3)^2-P_r(m_1+m_3)-P_ri\omega]}$; $D_{38} = \frac{P_rA(m_1+m_2)D_{10}}{[(1+R^2)(m_1+m_2)^2-P_r(m_1+m_2)-P_ri\omega]}$; $D_{39} = \frac{2P_rm_3m_8D_1D_{23}}{[(1+R^2)(m_3+m_8)^2-P_r(m_3+m_8)-P_ri\omega]}$; $D_{40} = \frac{2P_rm_3m_8D_1D_{23}}{[(1+R^2)(m_2+m_8)^2-P_r(m_2+m_8)-P_ri\omega]}$; $D_{41} = \frac{2P_rm_2m_8D_2D_{23}}{[(1+R^2)(m_1+m_8)^2-P_r(m_1+m_8)-P_ri\omega]}$; $D_{42} = \frac{2P_rm_3m_7D_1D_{24}}{[(1+R^2)(m_3+m_7)^2-P_r(m_3+m_7)-P_ri\omega]}$; $\frac{2P_rm_2m_7D_2D_{24}}{[(1+R^2)(m_2+m_7)^2-P_r(m_2+m_7)-P_ri\omega]} =$

$D_{43}; \frac{2P_rm_1m_7D_3D_{24}}{[(1+R^2)(m_1+m_7)^2-P_r(m_1+m_7)-P_ri\omega]} = D_{44}; \frac{2P_rm_3m_6D_1D_{26}}{[(1+R^2)(m_3+m_6)^2-P_r(m_3+m_6)-P_ri\omega]} =$

$D_{45}; \frac{2P_rm_2m_6D_2D_{26}}{[(1+R^2)(m_2+m_6)^2-P_r(m_2+m_6)-P_ri\omega]} = D_{46}; \frac{2P_rm_1m_6D_3D_{26}}{[(1+R^2)(m_1+m_6)^2-P_r(m_1+m_6)-P_ri\omega]} =$

$D_{47}; \frac{2P_rm_3^2D_1D_{28}}{[4(1+R^2)m_3^2-2P_rm_3-P_ri\omega]} = D_{48}; \frac{2P_rm_2m_3D_2D_{28}}{[(1+R^2)(m_2+m_3)^2-P_r(m_2+m_3)-P_ri\omega]} =$

$D_{49}; \frac{2P_rm_1m_3D_3D_{28}}{[(R^2)(m_1+m_3)^2-P_r(m_1+m_3)-P_ri\omega]} = D_{50}; \frac{2P_rm_2m_3L_1D_1}{[(1+R^2)(m_2+m_3)^2-P_r(m_2+m_3)-P_ri\omega]} =$

$D_{51}; \frac{2P_r m_2^2 L_1 D_2}{[4(1+R^2)m_2^2 - 2P_r m_2 - P_r i\omega]} = D_{52}; \frac{2P_r m_1 m_2 L_1 D_3}{[(1+R^2)(m_1+m_2)^2 - P_r(m_1+m_2) - P_r i\omega]} =$
$D_{53}; \frac{2P_r m_1 m_3 L_2 D_1}{[(1+R^2)(m_1+m_3)^2 - P_r(m_1+m_3) - P_r i\omega]} = D_{54}; \frac{2P_r m_1 m_2 L_2 D_2}{[(1+R^2)(m_1+m_2)^2 - P_r(m_1+m_2) - P_r i\omega]} = D_{55};\ D_{56} =$
$\frac{2P_r m_1^2 L_2 D_3}{[4(1+R^2)m_1^2 - 2P_r m_1 - P_r i\omega]}$

$D_{57} = D_{66} - D_{58} + L_9 - L_{10} - L_{11} + L_{12} - L_{13} + L_{14} + L_{15} - D_{74} - D_{75} + D_{76} + D_{77} - D_{78} - D_{79} + D_{80} - D_{81} - D_{82}$

$\frac{m_5 A D_{11}}{[m_5^2 - m_5 - N_2]} = D_{58}; \frac{m_4 A D_{12}}{[m_4^2 - m_4 - N_2]} = D_{59}; \frac{2m_3 A D_{13}}{[m_3^2 - m_3 - N_2]} = D_{60}; \frac{2m_2 A D_{14}}{[m_2^2 - m_2 - N_2]} = D_{61}; \frac{2m_1 A D_{15}}{[m_1^2 - m_1 - N_2]} =$
$D_{62}; \frac{(m_2+m_3) A D_{16}}{[(m_2+m_3)^2 - (m_2+m_3) - N_2]} = D_{63}; \frac{(m_1+m_3) A D_{17}}{[(m_1+m_3)^2 - (m_1+m_3) - N_2]} = D_{64}; \frac{(m_1+m_2) A D_{18}}{[(m_1+m_2)^2 - (m_1+m_2) - N_2]} =$
$D_{65}; \frac{Gr_T D_{31} cos\propto}{[m_9^2 - m_9 - N_2]} = D_{66}; \frac{Gr_T D_{32} cos\propto}{[m_4^2 - m_4 - N_2]} = D_{67}; \frac{Gr_T L_3 cos\propto}{[m_3^2 - m_3 - N_2]} = D_{68}; \frac{Gr_T L_4 cos\propto}{[m_2^2 - m_2 - N_2]} =$
$D_{69}; \frac{Gr_T L_5 cos\propto}{[m_1^2 - m_1 - N_2]} = D_{70}; \frac{Gr_T L_6 cos\propto}{[(m_2+m_3)^2 - (m_2+m_3) - N_2]} = D_{71}; \frac{Gr_T L_7 cos\propto}{[(m_1+m_3)^2 - (m_1+m_3) - N_2]} =$
$D_{72}; \frac{Gr_T L_8 cos\propto}{[(m_1+m_2)^2 - (m_1+m_2) - N_2]} = D_{73}; \frac{Gr_T D_{39} cos\propto}{[(m_3+m_8)^2 - (m_3+m_8) - N_2]} = D_{74}; \frac{Gr_T D_{40} cos\propto}{[(m_2+m_8)^2 - (m_2+m_8) - N_2]} =$
$D_{75}; \frac{Gr_T D_{41} cos\propto}{[(m_1+m_8)^2 - (m_1+m_8) - N_2]} = D_{76}; \frac{Gr_T D_{42} cos\propto}{[(m_3+m_7)^2 - (m_3+m_7) - N_2]} = D_{77}; \frac{Gr_T D_{43} cos\propto}{[(m_2+m_7)^2 - (m_2+m_7) - N_2]} =$
$D_{78}; \frac{Gr_T D_{44} cos\propto}{[(m_1+m_7)^2 - (m_1+m_7) - N_2]} = D_{79}; \frac{Gr_T D_{45} cos\propto}{[(m_3+m_6)^2 - (m_3+m_6) - N_2]} = D_{80}; \frac{Gr_T D_{46} cos\propto}{[(m_2+m_6)^2 - (m_2+m_6) - N_2]} =$
$D_{81}; \frac{Gr_T D_{47} cos\propto}{[(m_1+m_6)^2 - (m_1+m_6) - N_2]} = D_{82}$

$L_1 = D_{25} + D_{29};\ L_2 = D_{27} + D_{30}; L_3 = D_{33} + D_{48};\ L_4 = D_{34} + D_{52};\ L_5 = D_{35} + D_{56};\ L_6 = D_{36} + D_{49} + D_{51};\ L_7 = D_{37} + D_{50} + D_{54};\ L_8 = D_{38} + D_{53} + D_{55};\ L_9 = [D_{59} + D_{67}]; L_{10} = [D_{60} + D_{68}];\ L_{11} = [D_{61} + D_{69}]; L_{12} = [D_{62} + D_{70}]; L_{13} = [D_{63} + D_{71}]; L_{14} = [D_{64} + D_{72}]; L_{15} = [D_{65} + D_{73}]$